EIGEN-ANALYSIS OF AN INVISCID CHANNEL FLOW WITH A FINITE FLEXIBLE PLATE IN ONE WALL

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1 Introduction

The aim of the present study is to determine the stability of a flexible plate—or compliant wall—that comprises a section of one wall of a two-dimensional channel in which there is a uniform (inviscid) flow; the system is depicted in Fig. 1. This problem may be considered an extension of the large body of work on the behaviour of flexible plates and compliant walls in open flows; e.g. see [1, 9, 11]. For these it is well known that a flexible plate first loses its stability to divergence, and at higher applied flow speeds to a flutter-type instability caused by modal coalescence. The closely related problem of (inviscid) fluid-conveying two-dimensional flexible pipes for which symmetric (sinuous) deformation of both walls occur has been studied using a quasi one-dimensional plug-type flow. Pipe buckling (divergence) and flutter are predicted at sufficiently high flow speeds; e.g. see [5, 6]. Viscous fluid-flow models for the the main configuration of Fig. 1 have been developed for linear deformations of the flexible insert [2–4] and for large-amplitude deformations of a membrane [8] demonstrating a range of instabilities that include divergence, travelling wave flutter, Tollmien-Schlichting waves and nonlinear self-excited oscillations. However, these complex models for the present system are restricted to unsteady laminar flow at low Reynolds number. The results of the present study may be considered as being relevant to very high (infinite) Reynolds-number flows that may be more appropriate in both engineered and some biological systems. Our goal is to determine the effect of channel height on the stability of the compliant channel wall.

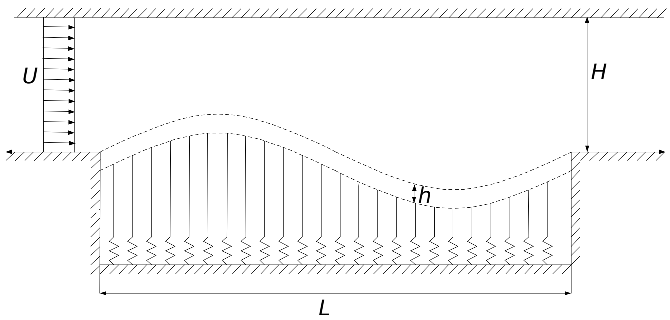

Figure 1: Schematic of the fluid-structure interaction system; the uniformly distibuted spring foundation is omitted for a simple elastic plate.

2 Method

The motion of the flexible plate of length L is described by the one dimensional beam equation and is modelled using a finite difference method, where it is discretised into N mass points. Hinged-hinged boundary conditions are applied to the plate ends.

A potential flow is assumed, giving a perturbation which satisfies Laplace's equation, and the pressure obtained from the unsteady Bernoulli equation. The flow-solution is found using a linearised unsteady boundary-element method, constructed using source/sink singularites, following the linear theory developed by [7] in which the singularities on the compliant surface do not move with the flexible plate but stay fixed on the undisturbed plane. In addition, boundary-elements are used on the rigid upper and lower walls of the channel to enforce the no-flux condition on these boundaries.

A fully-coupled system is developed by matching the pressure at the fluid-solid boundary and a single matrix equation is derived. Using the state-space method developed by [9] the eigenvalues s and eigenvectors W of this system are extracted where $s = s_{\mathrm{R}} + i\, s_{\mathrm{I}}$ is the complex frequency of the system. From the eigenvalue analysis instability onset is predicted.

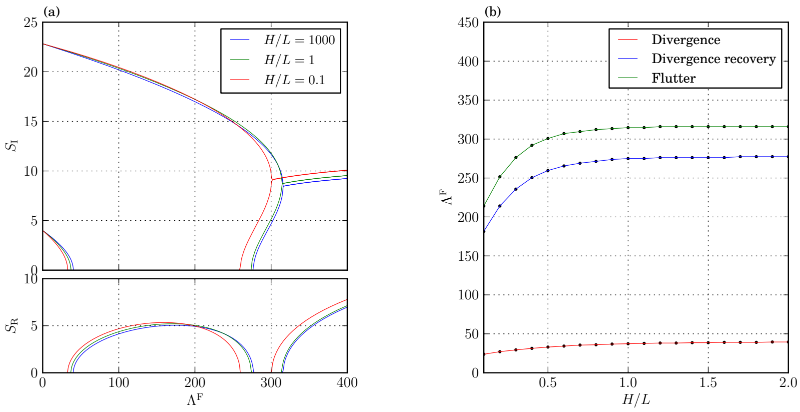

Figure 2: (a) The imaginary (top) and real (bottom) eigenvalues of the first two modes for three different channel heights; (b) dependence of divergence, divergence recovery and flutter onset on channel height.

3 Results and Discussion

Three cases are considered: (i) A simple elastic plate, (ii) a flexible plate with structural damping and (iii) a spring-backed flexible plate. Fig. 2(a) is a typical result showing the variation of system eigenvalues with the stiffness ratio, $\Lambda^{\mathrm{F}} = \rho_{\mathrm{f}} U^2 L^3 / B$, for case (i) for three different nondimensional channel heights and where the mass ratio, $\mu = \rho_{\mathrm{f}} L / \rho_{\mathrm{w}} h$ is 154. In the foregoing ρ_{f} and ρ_{w} are respectively the fluid and plate-material densities, U is the flow velocity, L is the flexible plate length and B and h are respectively its flexural rigidity and thickness. By keeping the system dimensions constant, an increase in Λ^{F} effects an increase in U. For nondimensional channel height $H/L = 0.1$ divergence onset, divergence recovery and modal-coalescence flutter are respectively seen to occur at $\Lambda^{\mathrm{F}} = 33$, 260 and 300. Fig. 2(b) shows the effect of H/L on the Λ^{F} of these landmark values. These results show that reducing H/L tends to destabilise the system by causing instability onset at lower flow velocities. This is caused by a Bernoulli-type effect for which a narrower channel causes an intensification of the pressure-perturbation on the flexible plate. It can also be seen in Fig. 2(b) that the critical values of Λ^{F} approach the value for the case with no upper wall and $H/L = 1.5$ gives a good approximation for $H/L = \infty$. When $H/L = 1.5$, the critical stiffness-ratio for divergence onset is $\Lambda^{\mathrm{F}}_{\mathrm{d}} \approx 40$ which agrees with [10]. Fig. 2(a) also shows that the morphology of the eigen-solutions is slightly different to the open-flow results. Instead of a distinct point at which coalescence of modes one and two occurs, they now converge to the same value. This behaviour is similar to that when small levels of damping are included in the corresponding open-flow system [9].

For the spring-backed flexible plate, the value of divergence onset for $H/L = 1.5$ again agrees with [10], and also with the travelling-wave-based analytical prediction of [1], because as the plate oscillates in a higher mode it becomes comparable with models of infinitely long flexible plates. As in the previous two cases, decreasing the channel width destablises the system, but for a spring-backed plate, H/L must be much lower before the upper wall has a significant effect; in fact the scaling of H is better matched to the critical wavlength of the disturbance than the overall length of the compliant wall. We have also developed an extension of the analytical prediction of [1] to account for the presence of the rigid upper wall. This again shows good agreement with the present eigen-analysis of finite-length compliant walls.

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